《Fundamentals of Computer Graphics》

Lecture 6、Color

Yong-Jin Liu

liuyongjin@tsinghua.edu.cn

Material by S.M.Lea (UNC)

Color Basics

- Color: made from three basic primaries: red, green, and blue.
  - This model is related to the manner in which the eye operates.
  - It is consistent with how graphics displays generate colors by mixing amounts of some built-in red, green, and blue colors.
- Color depends on subtle interactions between the physics of light radiation and the eye-brain system.

Issues in Color Theory

- How are colors described numerically?
- How do these descriptions relate to everyday ways of describing color?
- How can colors be compared?

Issues in Color Theory (2)

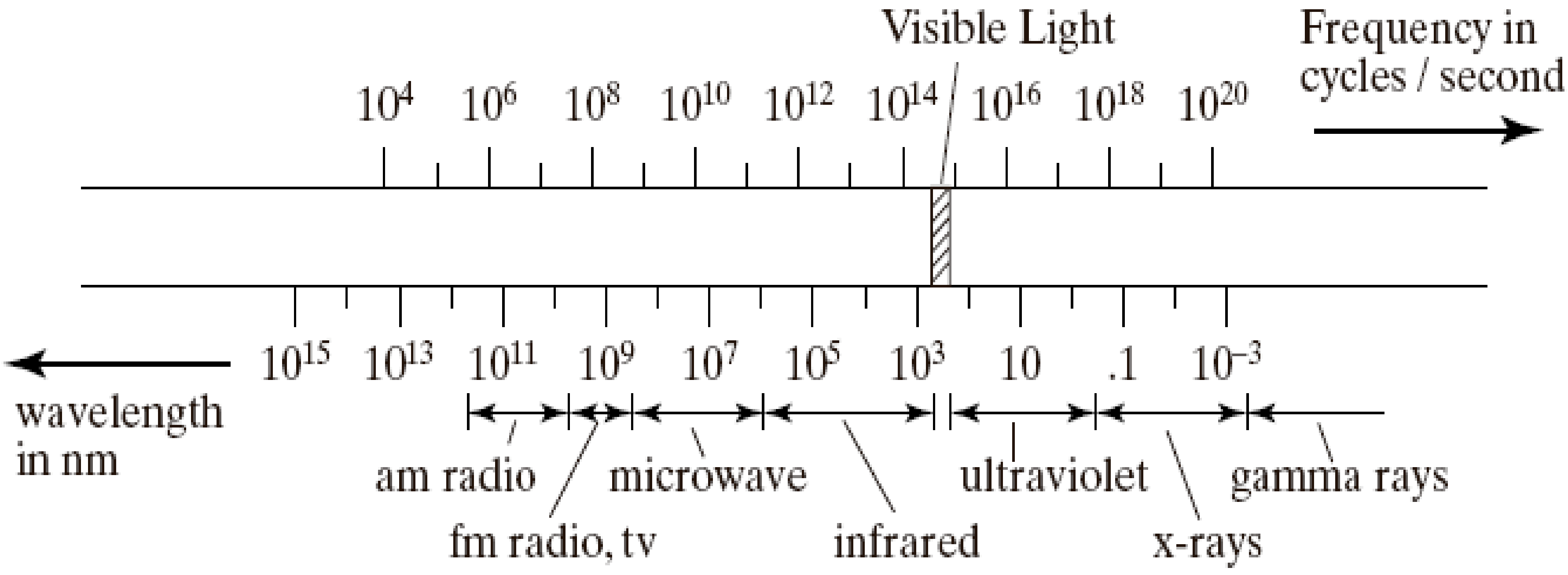
- What range of colors can a CRT display or a printed page exhibit?
- How can color lookup tables be loaded to produce the colors required?
- How do we deal with a range of color when a device can display only, say, 256 colors?

Light

- Light is an electromagnetic phenomenon, like television waves, infrared radiation, and x-rays.
- By light, we mean those waves that lie in a narrow band of ***wavelengths*** in the so-called *visible spectrum*.
- The wavelength is the distance light travels during one cycle of its vibration.
- Wavelength λ and frequency f are related by $\lambda = v/f$, where v is the speed of light in the medium of interest.
  - In air (or a vacuum) v = 300,000 km/sec; in glass, it is about 65 percent as fast.

Light (2)

- This figure shows the location of the visible spectrum (for humans) within the entire electromagnetic spectrum.



Light (3)

- In the figure, the frequency f increases to the right, whereas wavelength λ increases to the left.
- The eye responds to light with wavelengths between approximately 400 and 700 nm (nanometers).

The Human Eye

- The retina of the eye is its light-sensitive membrane. It lines the rear portion of the eye's wall and contains two kinds of receptor cells: cones and rods.
- The **cones** are the color-sensitive cells, each of which responds to a particular color, red, green, or blue.
- The **rods** cannot distinguish colors, nor can they see fine detail.

The Human Eye:Cones

- According to the tri-stimulus theory, the color we see is the result of our cones' relative responses to red, green, and blue light.
- The human eye can distinguish about 200 intensities of red, green, and blue, each.
- An eye has 6 to 7 million cones, concentrated in a small portion of the retina called the **fovea**.
- Each cone has its own nerve cell, allowing the eye to discern tiny details.
  - To see an object in detail, the eye looks directly at it in order to bring the image onto the fovea.

The Human Eye: Rods

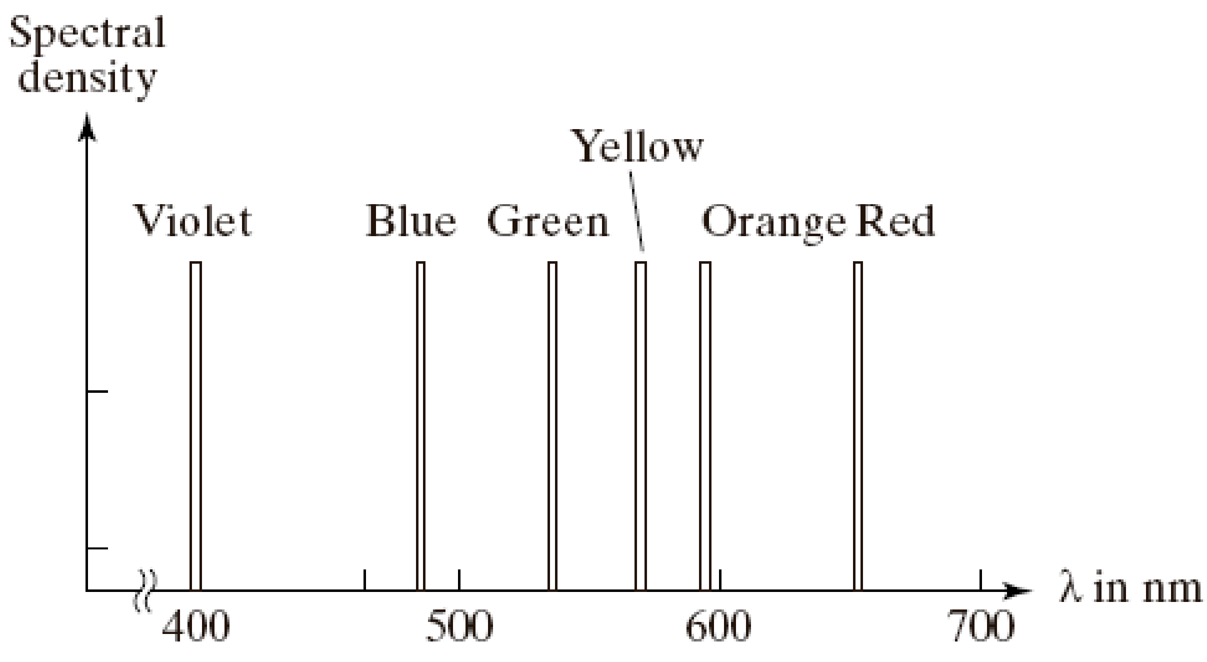
- Seventy-five million to 150 million rods are crowded onto the retina surrounding the fovea.
- A single nerve cell has many rods attached to it, preventing the discrimination of fine detail.
- Rods are very sensitive to low levels of light and can see things in dim light that the cones miss.
  - At night, for instance, it is best to look slightly away from an object so that the image falls outside the fovea.

Pure Light

- Some light sources, such as lasers, emit light of essentially a single wavelength or “pure spectral” light.
- We perceive 400 nm light as violet and 620 nm light as red, with the other pure colors lying in between these extremes.

Pure Light (2)

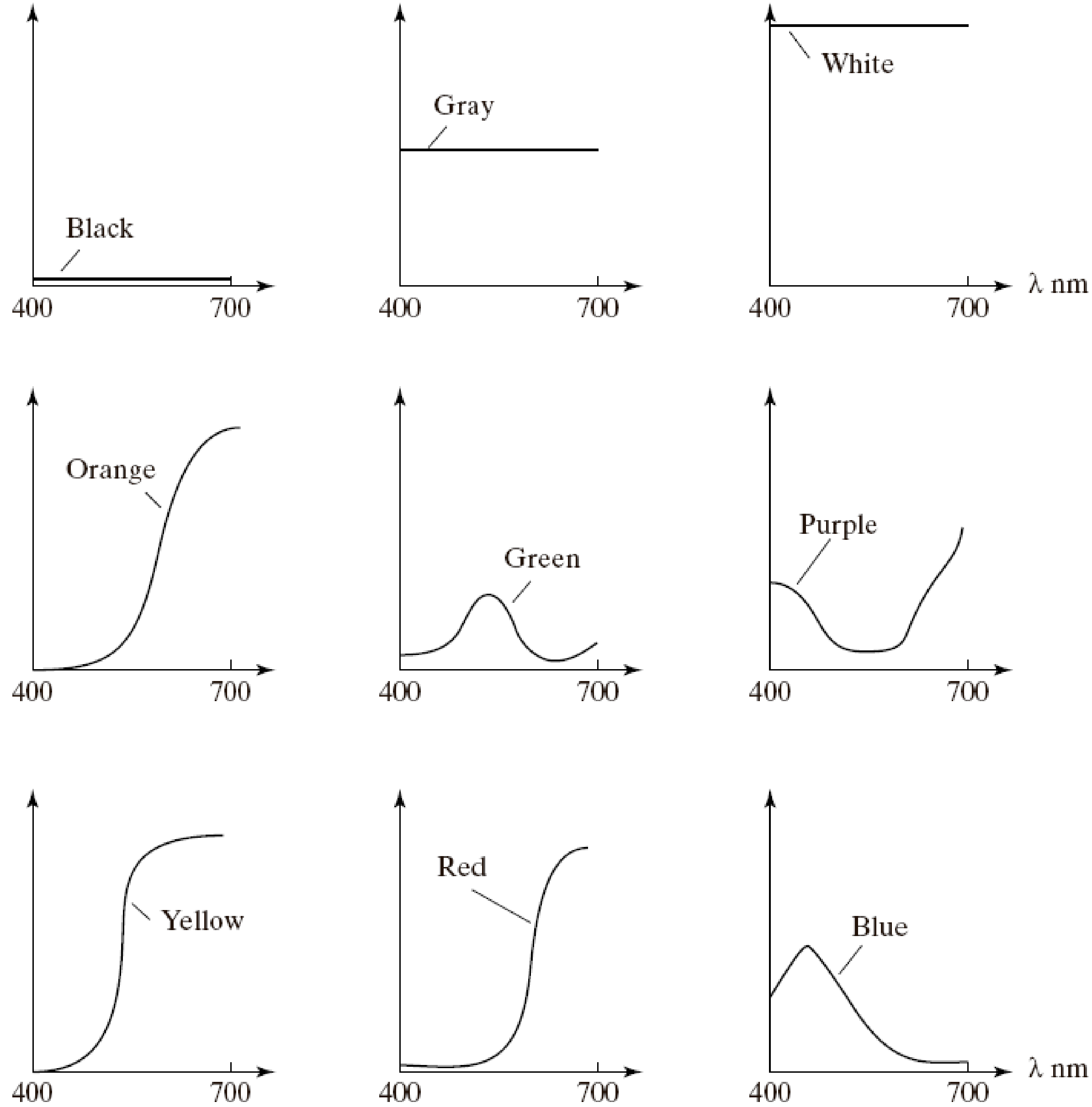
- The figure shows some example **spectral densities S**(λ) (power per unit wavelength) for pure lights and the common names given to the colors we see.



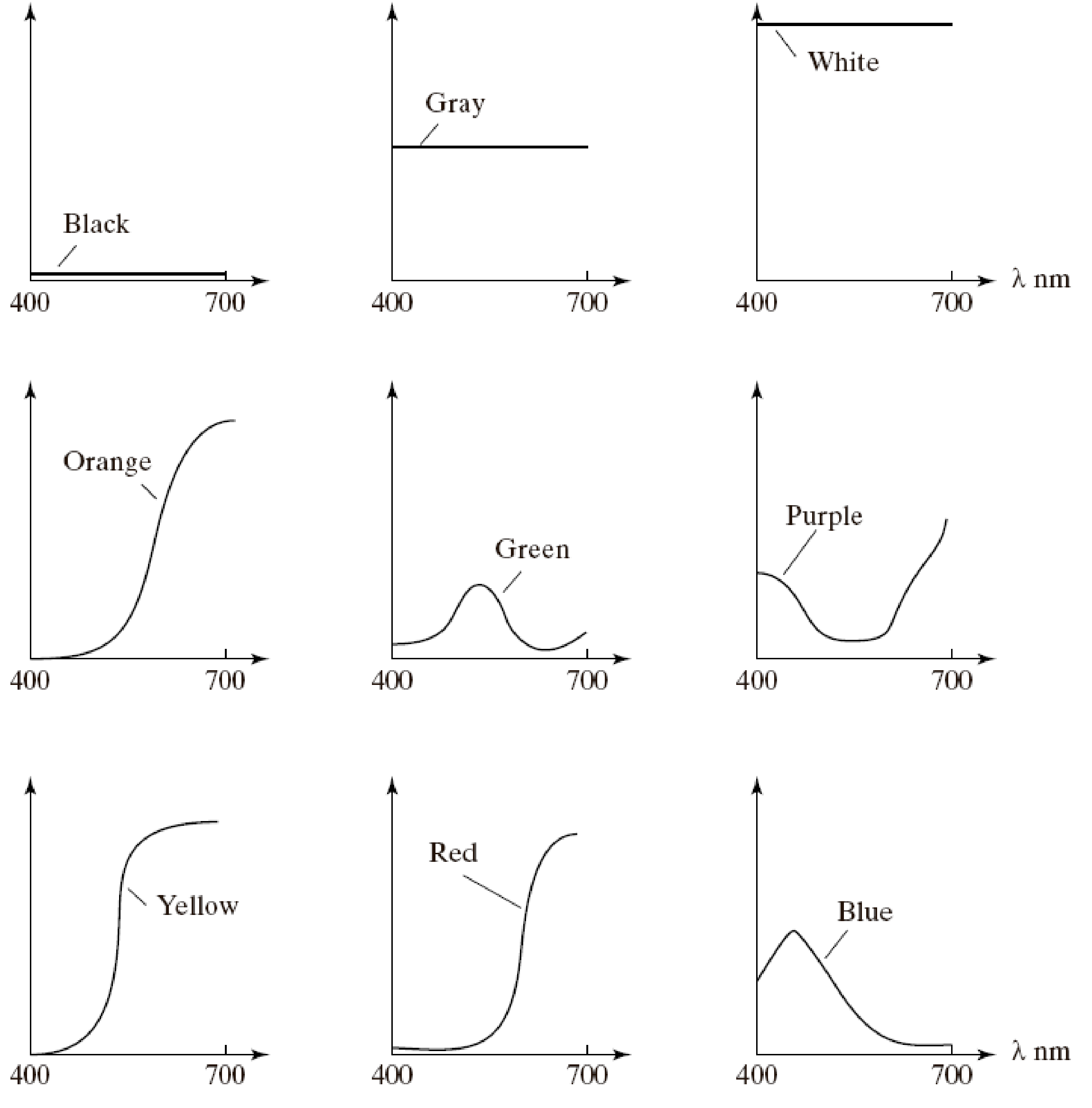
Light Sources

- The light from most sources does not consist of only one wavelength; instead, it contains varying amounts of power in a continuous set of wavelengths.
- Their set of spectral densities (or **spectra**) covers a band of wavelengths.
- The total power of the light in any band of wavelengths is found as the *area* under the density curve over that band.

Example Spectra for Common Colors



Example Spectra (2)



Light Sources (2)

- White light contains approximately equal amounts of power at all frequencies, as does gray (but at a lower intensity).
- Reds tend to have more power concentrated at the longer wavelengths, and blue at shorter wavelengths.
- Problem: *an enormous variety of spectral density functions is perceived by the eye as having the same color.*

RGB Color Blindness

- Some people's visual system cannot distinguish red and green light.
- Approximately 10% of males (but far fewer females) are "color-blind" in the sense that they cannot distinguish some colors from others.
- Tests have been devised to determine whether a given individual suffers from this lack of perception.

RGB Color Blindness (2)

- The companion web site illustrates such a test by placing a large number of circles in a large circle. (Can you see the numbers?)
- The small circles are rendered with slightly different colors, chosen so that a person with normal vision will see numerical patterns in the large circle but a color-blind person will see only a random pattern of circles.

Color Description

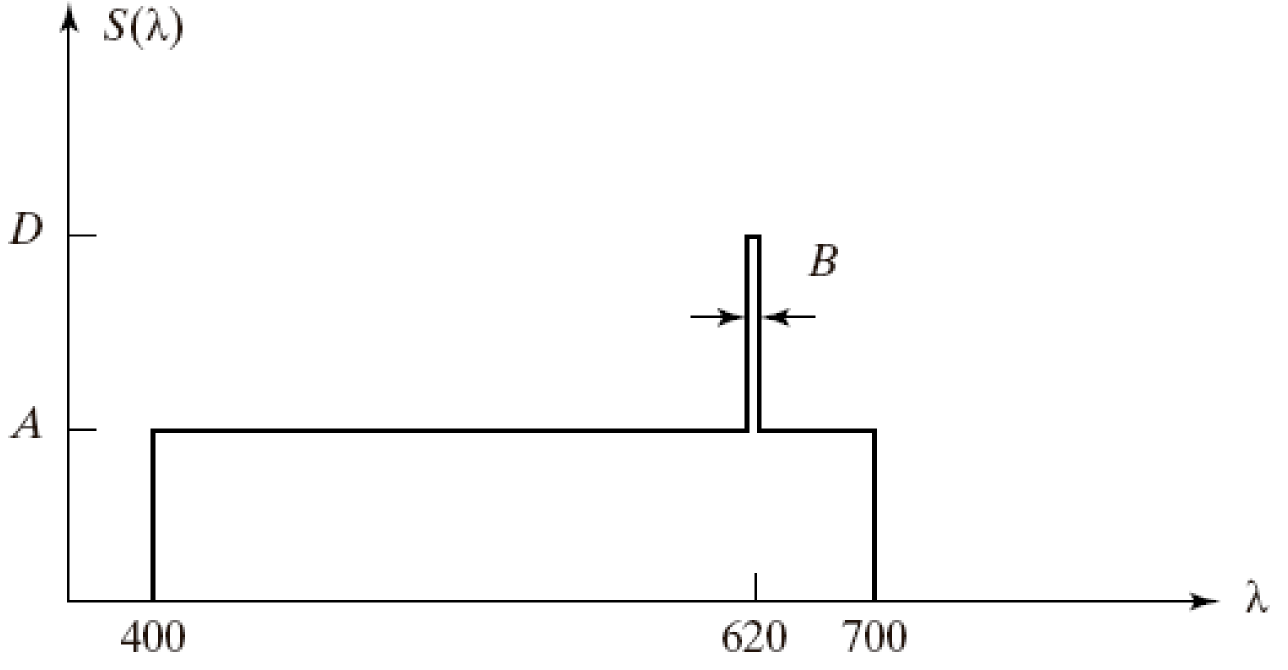
- Suppose that you want to describe a color precisely over the telephone, where all descriptions must be made orally.
- You want to describe the color by a small set of numbers.
- How many numbers are required?
  - Remarkably, the answer is three numbers: human **color perception** is three dimensional.

Color Description (2)

- What do the numbers mean?
- One simple way to describe a color capitalises on the variety of spectra that produce the same (perceived) color.

Color Description (3)

- It specifies a spectrum having the shape shown in the figure by three numbers: dominant wavelength, saturation, and luminance.



Color Description (4)

- The spectrum consists of a spike located at a dominant wavelength - 620 nm in the example.
- The location of the **dominant wavelength** specifies the **hue** of the color, in this case red.
- In addition, a certain amount of white light is present, represented by the rectangular pedestal that desaturates the light from a pure red, making it appear pink.

Color Description (5)

- The total power in the light (its **luminance**) is the area L under the entire spectrum.
- The rectangular shapes make the calculation simple: $L = (700 - 400)A + (D - A)B$.
- The **saturation** (or purity) of the light is defined as the percentage of luminance that is in the dominant component:

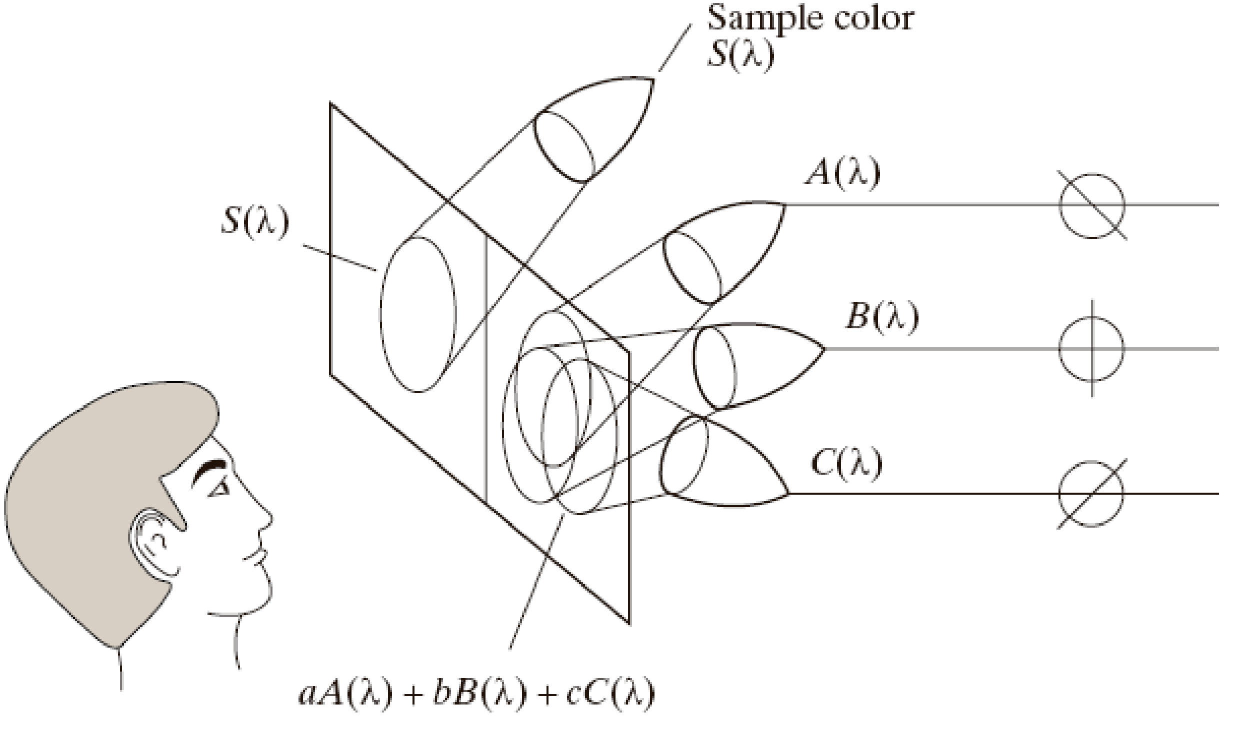
$$purity = \frac{(D - A)B}{L} \times 100\%$$

Color Description (6)

- If $D = A$, the purity is 0, and white light is observed without any trace of red.
- If $A = 0$, no white light is present, and a pure red light is seen.
- Pastel colors contain a large amount of white and are said to be **unsaturated**.
  - When two colors differ only in hue, the eye can distinguish about 128 different hues.
  - When two colors differ only in saturation, the eye can distinguish about 20 different saturations, depending on the hue.

Color Perception and Matching

- Colors are often described by comparing them with a standard color samples or lights and finding the closest match.



Color Perception and Matching (2)

- The sample color [spectral density $S(\lambda)$] is projected onto one part of a screen.
- The other part of the screen is the superposition of three test lights [spectral densities $A(\lambda)$, $B(\lambda)$, and $C(\lambda)$].
- The observer adjusts the intensities (a, b, and c) of the test lights until the test color $T(\lambda)=aA(\lambda)+bB(\lambda)+cC(\lambda)$ is indistinguishable from the sample color.
  - The two spectra, $S(\lambda)$ and $T(\lambda)$, may be quite different.
- If we say that the sample color is a sum of the amounts a, b, and c of the three test colors, what does this mean?

Vector Algebra of Colors

- Suppose that two spectral shapes, $S(\lambda)$ and $P(\lambda)$, have the same (perceived) color, denoted as $S = P$.
- Now add a third color, N, to both of these, by superposing light with spectrum $N(\lambda)$ on both.
- It is an experimental fact that these two new colors will still be indistinguishable!

Vector Algebra of Colors (2)

- Along with the symbol = [two colors are indistinguishable], we define + for colors so that $S + N$ denotes the color observed when the spectra $S(\lambda)$ and $N(\lambda)$ are added.
- This experimental fact can then be written as: if $(S = P)$ then $(N + S = N + P)$.
- The same goes for *scaling* colors [scaling their spectral densities or overall brightness]: If $S = P$, then $aS = aP$ for any (positive) scalar a.
- And it is meaningful to write *linear combinations* of two colors, A and B, as in $T = aA + bB$, where a and b are scalars.

Vector Algebra of Colors (3)

- Human color perception is *three dimensional:* any color C can be defined as the superposition of three primary colors, say R, G, and B: $C = rR + gG + bB$.
  - r, g, and b are scalars describing the amounts of each of the primaries contained in C.
- The symbols R, G, and B are suggestive of red, green, and blue, which are often used as the primaries because of the high sensitivity of cones to these three colors.
- But $C = rR + gG + bB$ works with any choice of primaries as long as one of them is not just a combination of the other two.

Vector Algebra of Colors (4)

- Given a set of three primary colors, *R*, *G*, and *B*, any other color, $C = rR + gG + bB$, can be represented in three-dimensional space by the point (*r*, g, *b*).
- For instance, if *R*, *G*, and *B* correspond to some versions of what we normally call red, green, and blue, then (0, 1, 0) will be perceived as a pure green of unit brightness and (.2, .3, .5) will be perceived as a yellow.
- If we double each component, we will obtain a color that is twice as bright but appears as the same color.

Tristimulus Theory

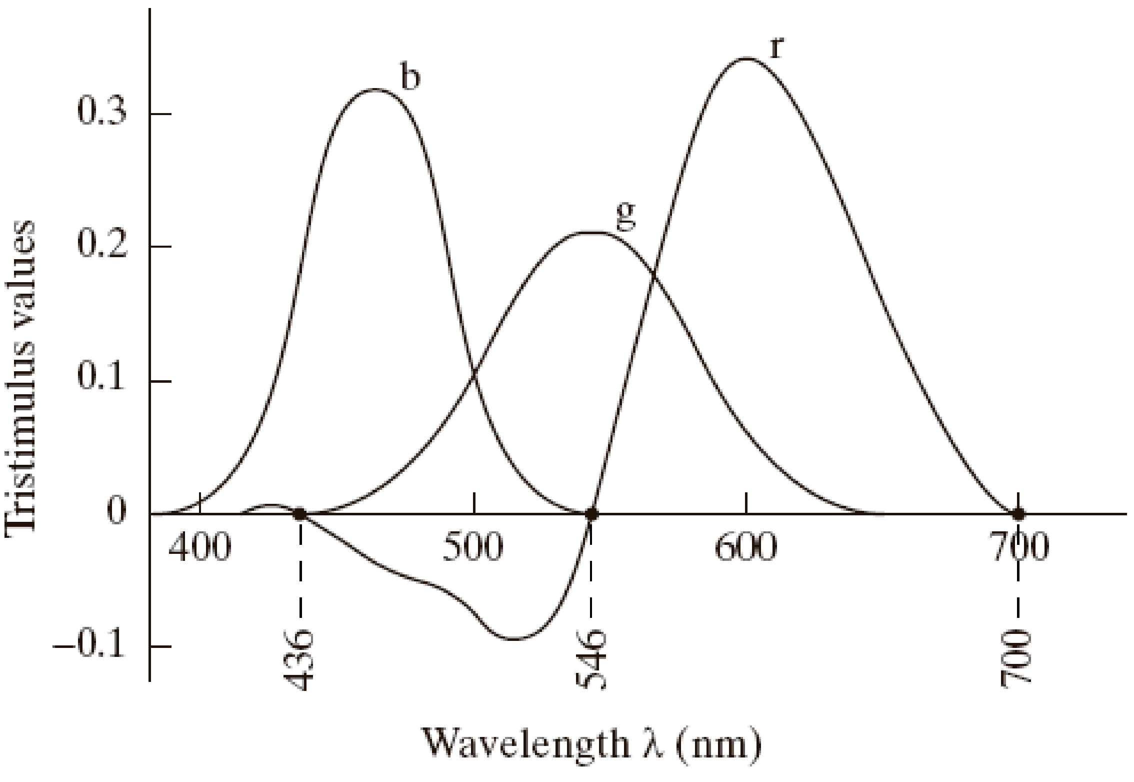
- Experiments have been run to see how people match colors.
- Of particular interest is one that combines three specific choices of *R*, *G*, and *B* in order to produce a (perceived) **pure spectral color** of wavelength, which is a totally saturated (no white) monochromatic color having its power concentrated at a single wavelength λ.

Tristimulus Theory (2)

- The primaries used were pure monochromatic red, green, and blue lights at wavelengths 700 nm, 546 nm, and 436 nm, respectively.
- The functions $r(\lambda)$, $g(\lambda)$, and $b(\lambda)$ show how much of these red, green, and blue lights are needed to match the pure spectral color at λ, $mono(\lambda) = r(\lambda)R + g(\lambda)G + b(\lambda)B$.

Tristimulus Theory (3)

- Example: a pure orange light *mono*(600) looks identical to the combination 0.37 R + .08 G.
- The spectrum of the orange light is not the same as the spectrum of this sum, but the two lights look exactly the same.



Tristimulus Theory (4)

- Problem: For this set of choices of *R*, *G*, and *B*, some of the scalars *r,* g, and *b* must be *negative* to make $C = rR + gG + bB$ correct!
- For instance, $r(\lambda)$ is negative at $\lambda = 520$.
- What is the physical meaning of the minus sign in a color such as $C = 0.7R + 0.5G - 0.2B$? Light that isn't there can't be removed.

Tristimulus Theory (5)

- Rewrite the equation as $C + 0.2B = 0.7R + 0.5G$.
- C alone cannot be constructed as the superposition of positive amounts of the primaries.
- The color $C + 0.2B$ can be matched by positive amounts of R and G.
- This is in fact what happens with any choice of visible primaries R, G, and B. Many colors can be fabricated (using positive coefficients r, g, and b), but some cannot, and one primary must be put on the other side of the equation.

Tristimulus Theory (6)

- The problem is that when two colors are added, the result is a less saturated color, and so it is impossible to form a highly saturated color by superposing two others.
- This is particularly obvious for any of the pure spectral colors, which are themselves saturated.

Scaling Color-matching Functions

- It is useful to scale the color matching functions so that they add to one.
- Define $$\bar{r}(\lambda) = \frac{r(\lambda)}{r(\lambda)+g(\lambda)+b(\lambda)}$$

$$\bar{g}(\lambda) = \frac{g(\lambda)}{r(\lambda)+g(\lambda)+b(\lambda)}$$

$$\bar{b}(\lambda) = \frac{b(\lambda)}{r(\lambda)+g(\lambda)+b(\lambda)}$$

$$\bar{r}(\lambda)+\bar{g}(\lambda)+\bar{b}(\lambda)=1$$

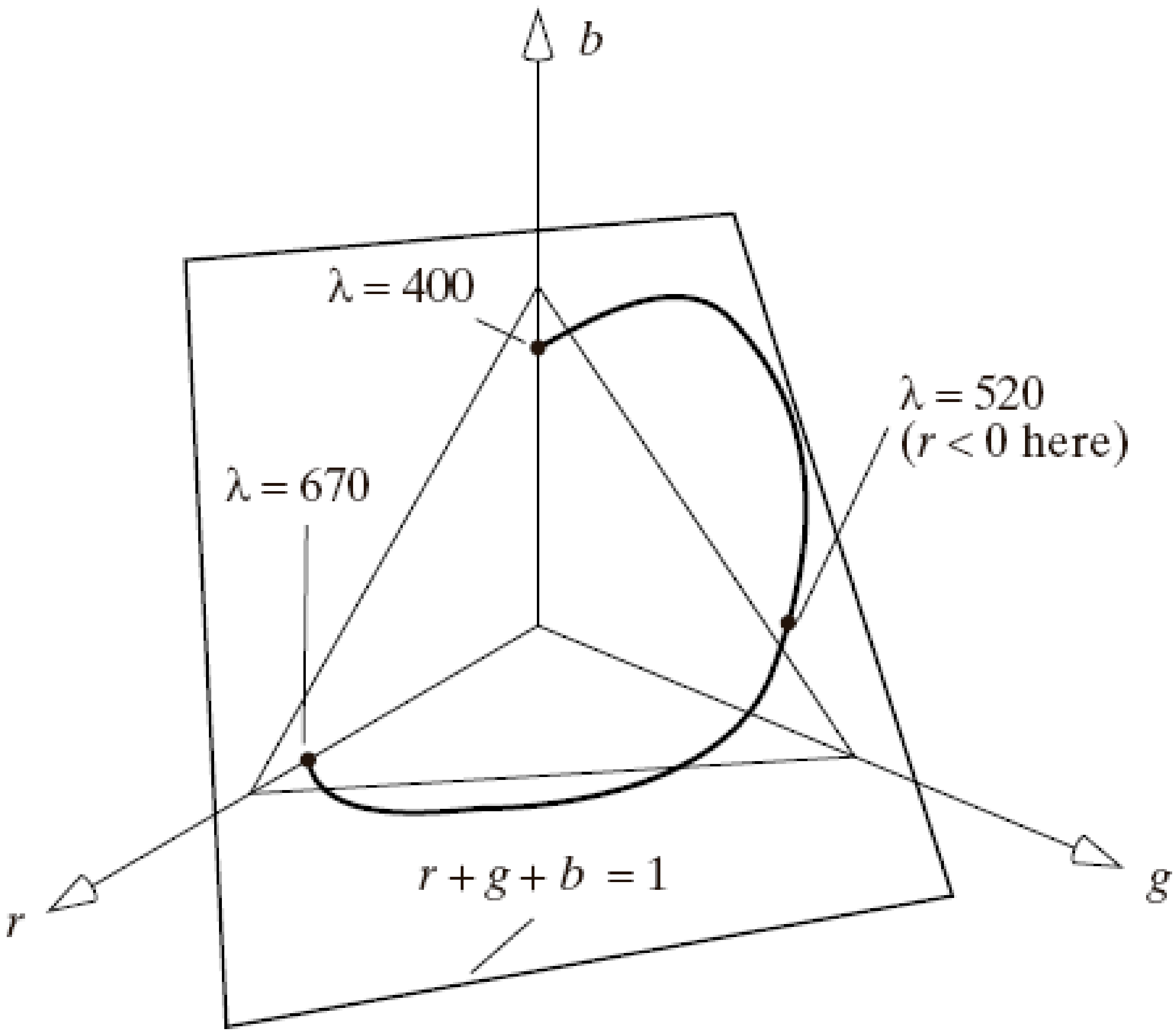
Scaling Color-matching Functions (2)

- The scaled values are called *chromaticity values* for *mono*(λ).
- They give the amounts of each of the primaries required to match a unit brightness light at λ.
- Removing brightness variations lets us specify colors with two numbers $(\bar{r}(\lambda), \bar{g}(\lambda))$ since $\bar{b}(\lambda) = 1 - \bar{r}(\lambda) - \bar{g}(\lambda)$

Scaling Color-matching Functions (3)

- We can plot the position of $(\overline{r}(\lambda), \overline{g}(\lambda), \overline{b}(\lambda))$ as λ varies across the visible spectrum.
- All points on the curve lie on the *r* + *g* + *b* = 1 plane.
- Because some coordinates are negative at certain values of λ, the curve does not lie totally inside the positive octant in this space.

Scaling Color-matching Functions (4)

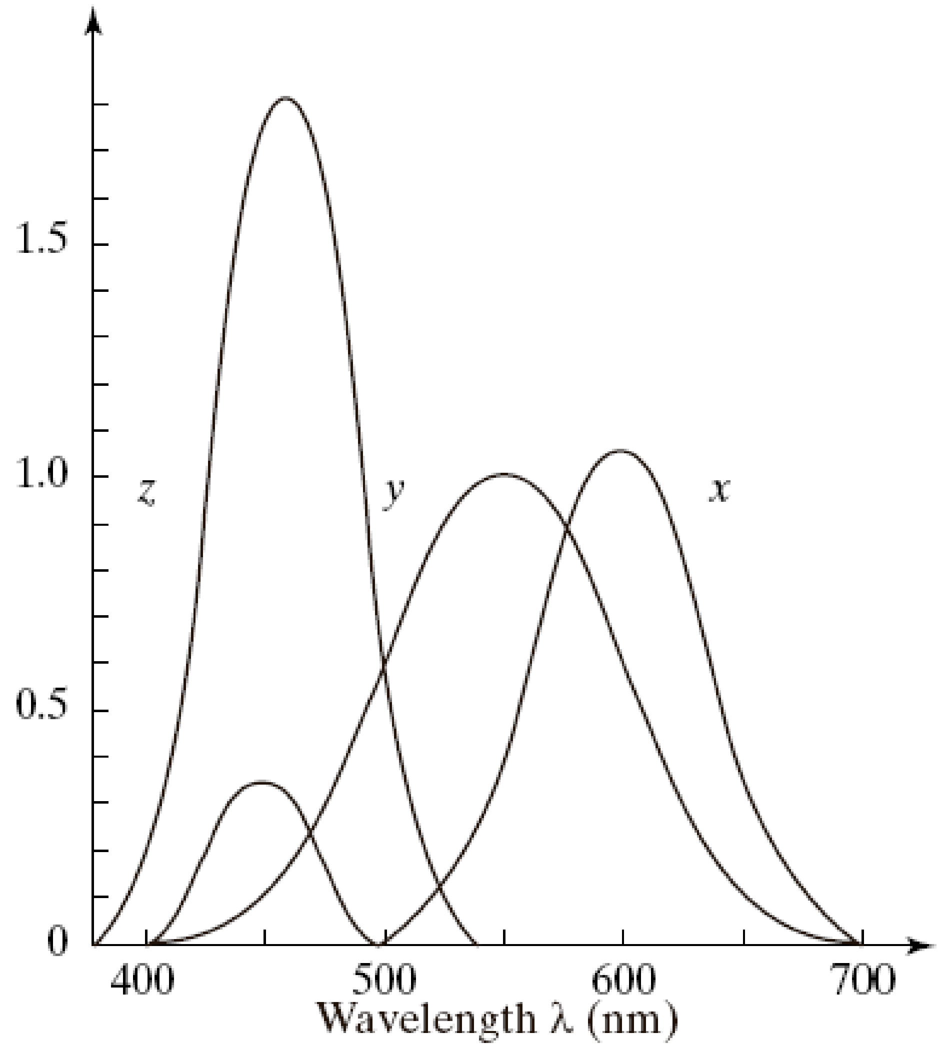


The CIE Standard

- A standard was devised in 1931 by the International Commission on Illumination (Commission Internationale de l’éclairage, or CIE).
- The CIE defined three special supersaturated primaries, X, Y, and Z.
- They don't correspond to real colors, but they do have the property that *all* real colors can be represented as *positive* combinations of them.

The CIE Standard (2)

- They are defined through color matching functions.
- Monochromatic light at wavelength λ is matched by a linear combination of these special primaries: mono(λ) = x(λ)X + y(λ)Y + z(λ)Z.



The CIE Standard (3)

- All three functions are positive at every λ, so *mono*(λ) is always a positive linear combination of the primaries.
- The *X*, *Y*, and *Z* primaries are defined by an affine transformation applied to color matching functions like *r*(λ), *g*(λ), and *b*(λ).
- *x*(λ), y(λ) and z(λ) are particular linear combinations of these shapes.

The CIE Standard (4)

- $y(\lambda)$ was chosen to have the same shape as the **luminous efficiency function:** the eye's response to monochromatic light of fixed strength at different wavelengths.
- This causes the amount of the *Y* primary present in a light to equal the overall intensity of the light.

The CIE Standard (5)

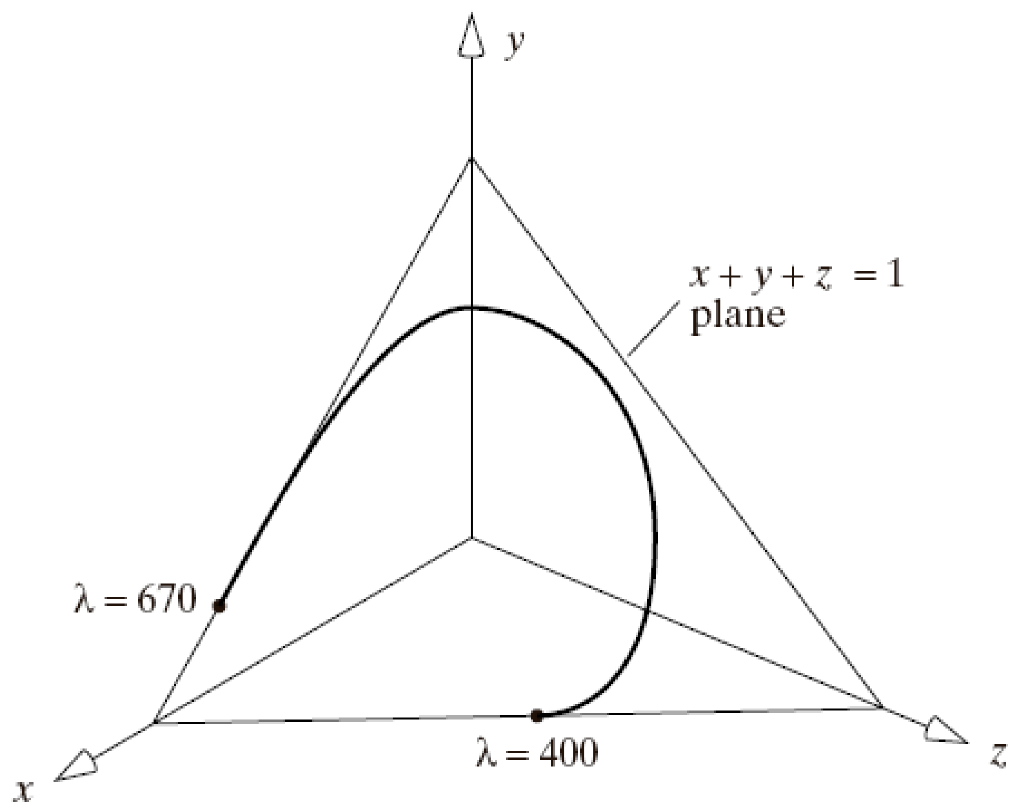
- We want normalized chromaticity values to maintain unit brightness:

$$\bar{x}(\lambda) = \frac{x(\lambda)}{x(\lambda) + y(\lambda) + z(\lambda)} \qquad \bar{y}(\lambda) = \frac{y(\lambda)}{x(\lambda) + y(\lambda) + z(\lambda)}$$

$$\bar{z}(\lambda) = \frac{z(\lambda)}{x(\lambda) + y(\lambda) + z(\lambda)} \qquad \bar{z}(\lambda) = 1 - \bar{x}(\lambda) - \bar{y}(\lambda)$$

The CIE Standard (6)

- The figure shows the curve $s(\lambda) = (\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda))$ lying in the positive octant of the xyz-plane.



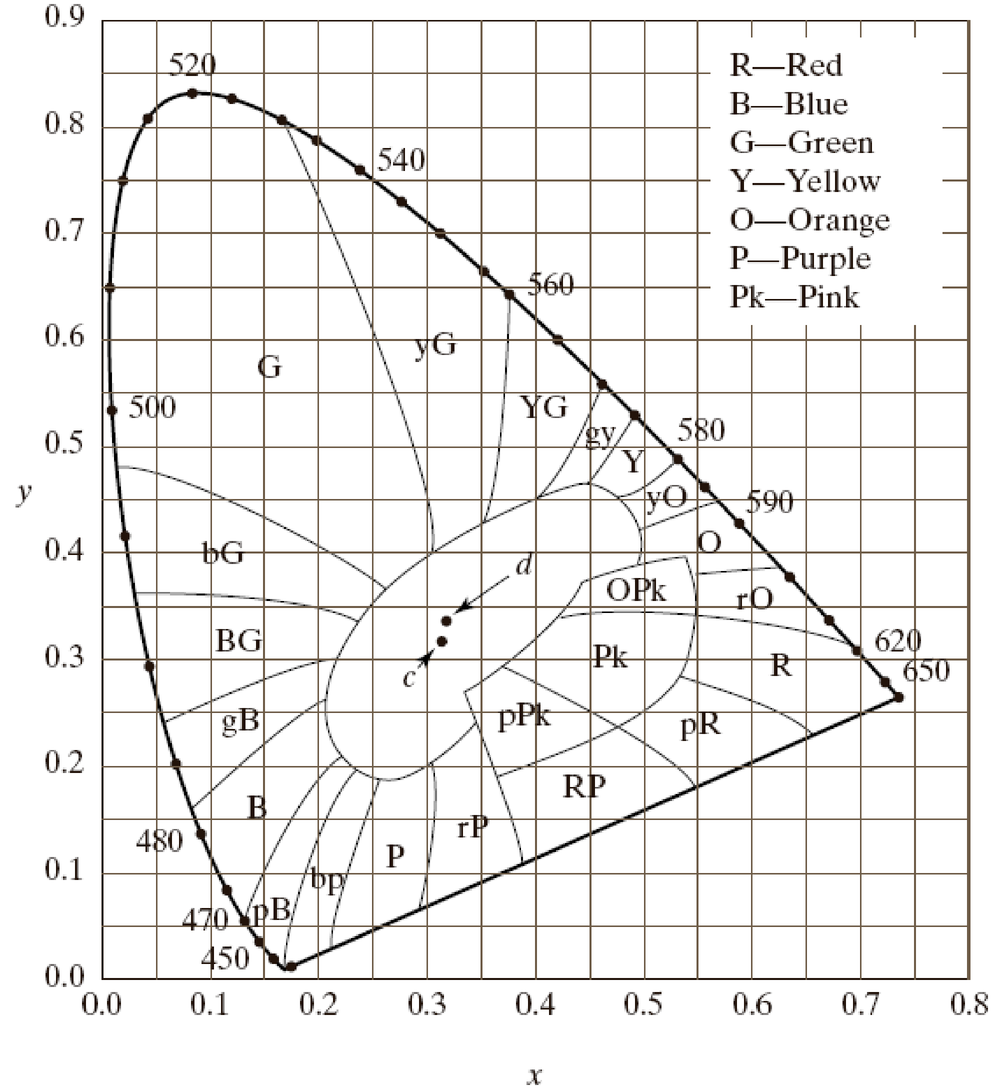
Constructing the CIE Chart

- The spectral color curve **s**(λ) lies in three-dimensional space, but because it lies on the $x + y + z = 1$ plane, it is easy to represent its shape in a two-dimensional chart that can be printed on a page for reference. Only x and y are needed to specify a (unit intensity) color because given (x, y) we can find z trivially (how?).

Constructing the CIE Chart (2)

- Thus the standard *CIE Chromaticity Diagram* is the curve (Fig. 11.10)

$$s'(\lambda) = (\bar{x}(\lambda), \bar{y}(\lambda))$$



Using the CIE Chart

- The diagram displays the horseshoe-shaped locus of all pure spectral colors, labelled according to wavelength.
- Inside the horseshoe lie all other visible colors.
- Points outside the horseshoe region do not correspond to visible light.

Using the CIE Chart (2)

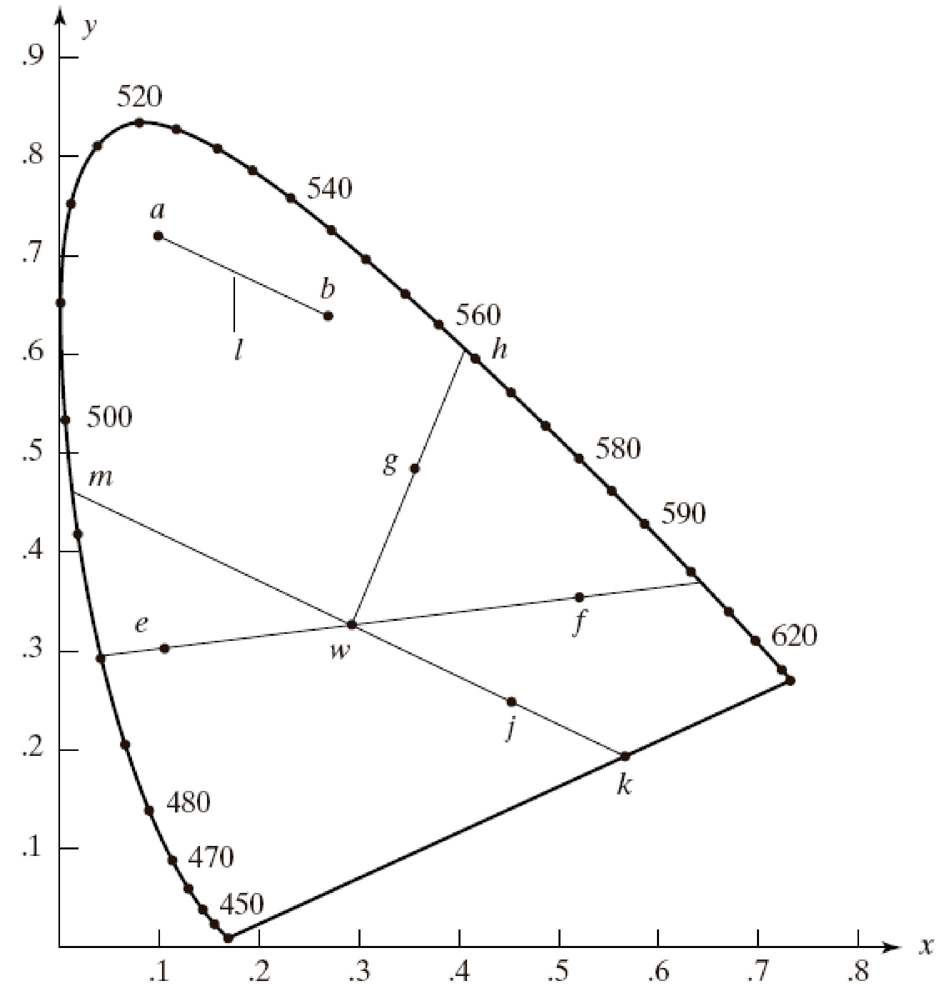
- Various regions are labelled with names that people commonly use to describe the colors found there.
  - For example, points near (0.6, 0.3) are perceived as red.
- Unfortunately, equal distances between points in the chart do not correspond to equal differences in perceived color.
  - Small changes in the *G* region cause only slight changes in perceived color.
  - Small changes in position in the *B* or *Y* regions cause large changes in perceived color.

Using the CIE Chart (3)

- Point *c* at $(x, y) = (0.310, 0.316)$ is a white color known as **Illuminant *C***, which is taken to be the fully unsaturated color.
  - It is often used as the reference color white in aligning some graphics monitors.
  - Illuminant C has the color of an overcast sky at midday.
- Point *d* at (0.313, 0.329) is the color that an ideal black-body radiator emits when raised to the "white-hot" temperature of 6504^{o} K. It is a little greener than Illuminant C.

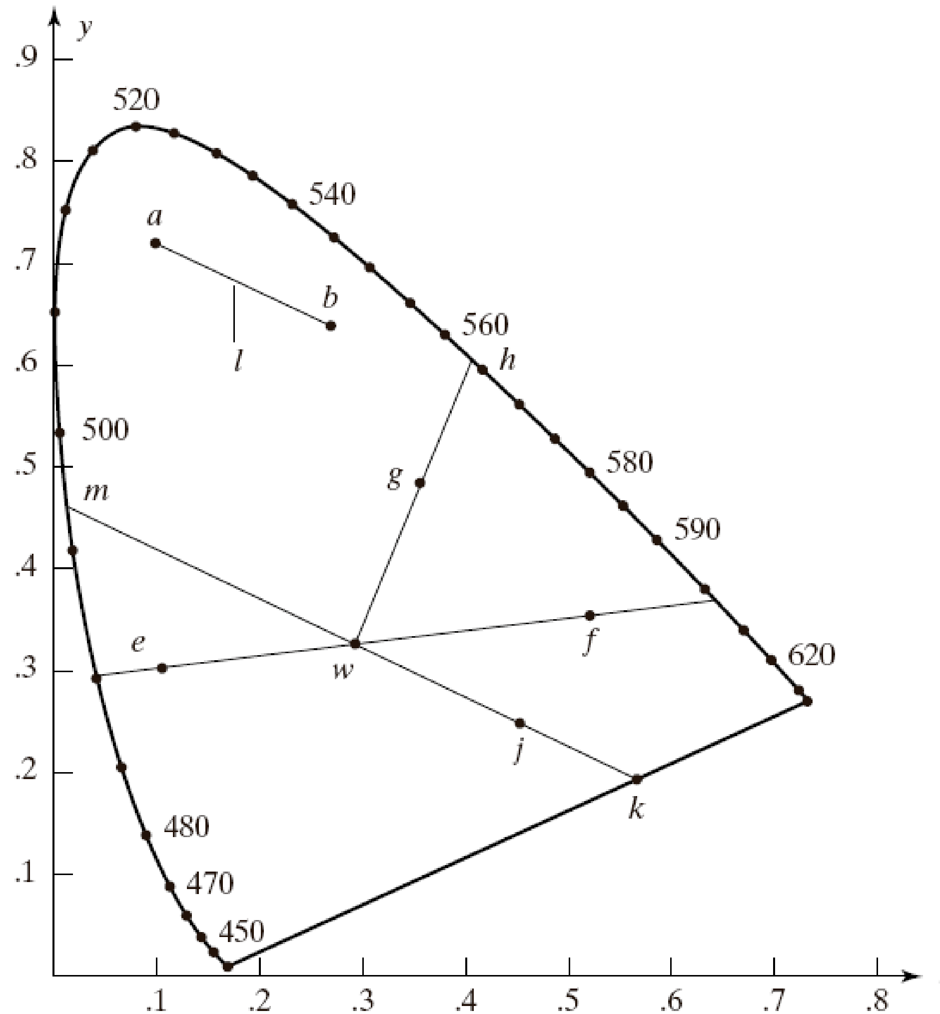
Using the CIE Chart (4)

- The CIE chromaticity diagram has many uses. Several of them stem from the ease with which we can interpret straight lines on the chart (Fig. 11.12).



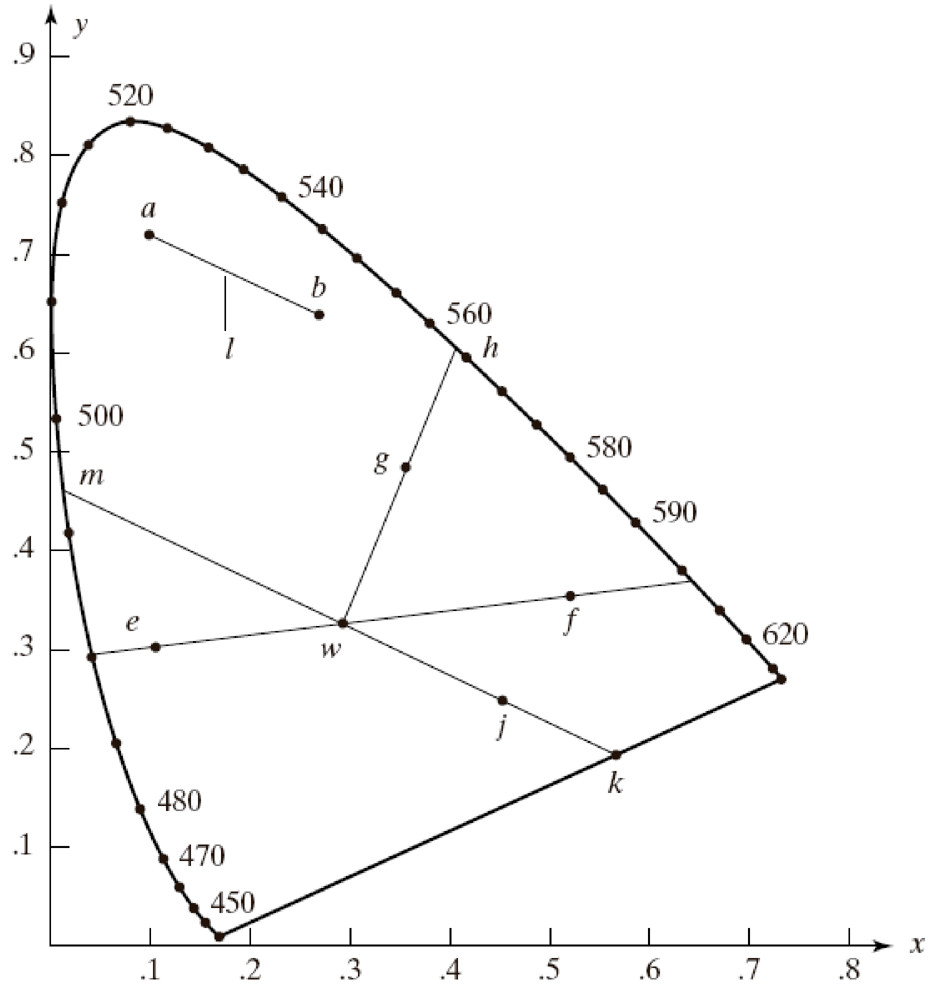
Using the CIE Chart (5)

- All points on line *l* between colors *a* and *b* are convex combinations of *a* and *b*, $(\alpha)a + (1 - \alpha)b$ for $0 \leq \alpha \leq 1$.
- Each point is a legitimate color; any color on the straight line (and only these) can be generated by shining various amounts of colors *a* and *b* onto a screen.



Using the CIE Chart (6)

- When two colors are added and their sum is white, we say the colors are **complementary.**
- *e* (blue-green) and *f* (orange-pink) are complementary colors because proper amounts of them added together form white, *w*.



Using the CIE Chart (7)

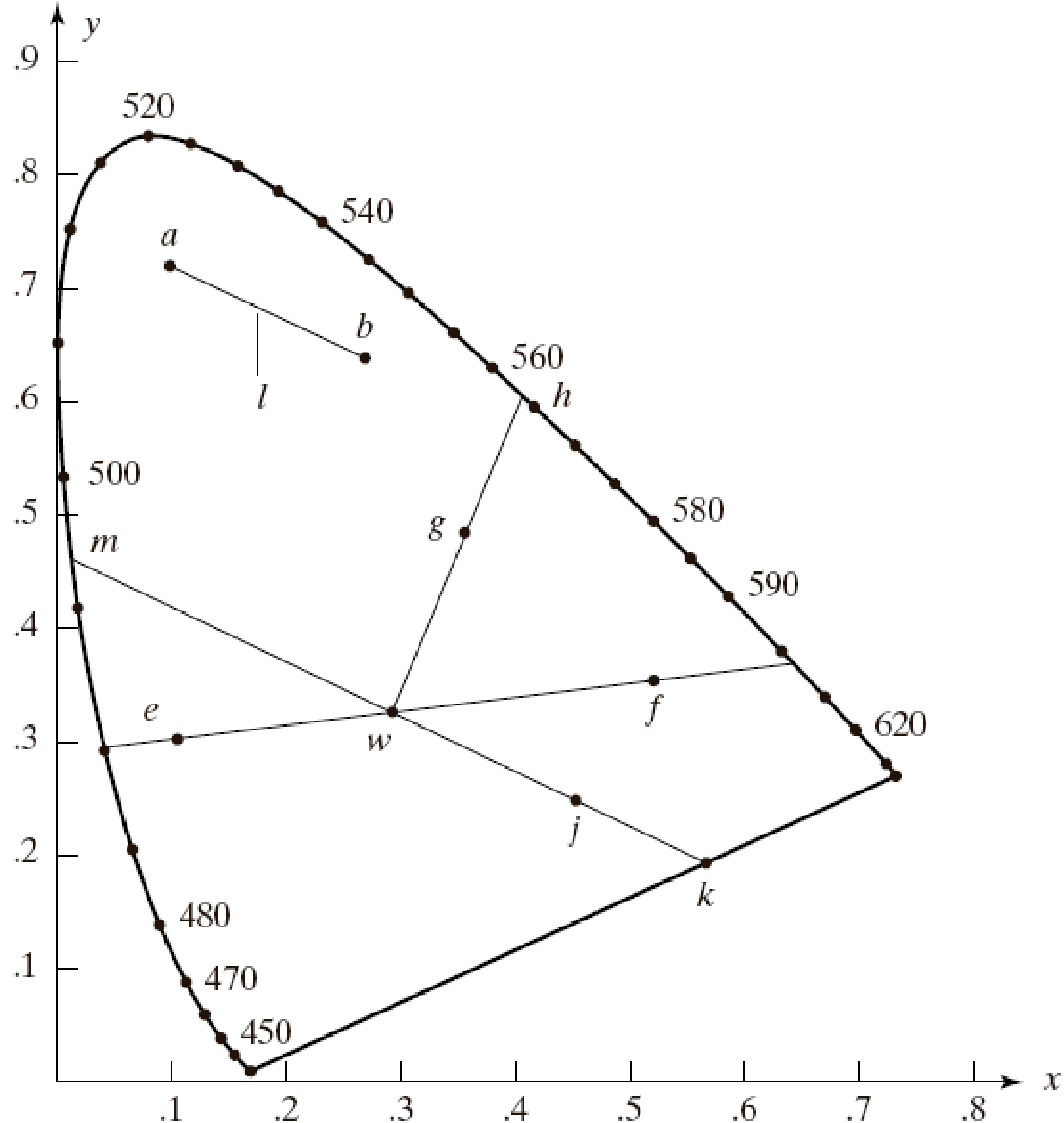
- Other complementary colors:
- red ↔ cyan
- green ↔ magenta
- blue ↔ yellow

Using the CIE Chart (8)

- Measuring dominant wavelength and purity of a given color such as *g* or j:
- *g* must be the linear combination of some pure spectral color (found on the edge of the horseshoe) and a standard white, *w*.
- To find which color, draw a line from *w* through *g* (to *h*) and measure the wavelength at *h:* 564 nm, a yellowish green.
- Similarly, the saturation or purity is just the ratio of distances *gw/hw.*

Using the CIE Chart (9)

- The color at *j* has no dominant wavelength: extending line *wj* hits *k* on the "purple line".
  - Colors along this line are combinations of red and violet rather than pure spectral colors.
- In such a case, the dominant wavelength is specified by finding the complement of *j* at *m* and using its wavelength with a *c* suffix, 498c.



y
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.8
x

Using the CIE Chart (11)

- *Color gamuts* are the range of colors that a device can produce.
- The printer gamut is the cloud; the monitor gamut is the triangle.
- Printers can produce some colors monitors cannot, and vice versa.