Lecture 3: Parametric curve and surface modeling

(1) Using the de Casteljau method with the three points (0,0), (2,4) and (6,1), locate P(t) for t-values 0, 0.2, 0.4, 0.6, 0.8 and 1.

(2) Write out the Bernstein function in Equation $\mathrm{B}_{\mathrm{k}}^{\mathrm{L}}(t) = \binom{L}{k}(1-t)^{L-k}t^{k}$ for the case L=4. Show that they are the terms one gets upon expanding $[(1-\mathrm{t})+\mathrm{t}]^4$. Repeat for L=5.

(3) Show that quadratic B-spline functions always sum to unity.

(4) Verify the formulas in Equation

$$\mathrm{N}_{0,4}(t) = \begin{cases} u(1-t) & for\ 0 \le t \le 1 \\ v(2-t) & for\ 1 \le t \le 2 \\ v(t-2) & for\ 2 \le t \le 3 \\ u(t-3) & for\ 3 \le t \le 4 \\ 0 & otherwise \end{cases}$$

for the cubic B-spline based on equispaced knots. Also calculate the first and second derivatives for the cubic B-spline and show that they are continuous everywhere.